Physics 195 / Applied Physics 195 — Assignment #8

Professor: Donhee Ham
Teaching Fellows: Brendan Deveney and Laura Adams
Date: Nov. 10, 2017
Due: 12:45pm + 10 min grace period, Nov. 21, 2017 at the dropbox outside Maxwell-Dworkin Room 131.

Problem 1 (200 pt): *pn* junction

(a) A silicon pn junction has $N_a = 10^{17}$ cm^{-3} on the p side, and $N_d = 10^{16}$ cm^{-3} on the n side. At 300K, calculate the chemical potential μ for each region, and draw the equilibrium band diagram. Find the built-in potential V_0 from the diagram.

(b) Show that the small-signal capacitance C of a semiconductor pn junction with doping densities N_a and N_d in the p and n regions with bias voltage V (which can be positive for forward bias and negative for reverse bias) is given by

$$C(V) = \frac{A}{2}\left[\frac{N_a N_d}{N_a + N_d} \times \frac{2e\epsilon}{V_0 - V}\right]^{1/2} \tag{1}$$

where A is the cross-sectional area of the pn-junction, ϵ is the electric permittivity of the semiconductor, and V_0 is the built-in potential. This problem involves solving the electrostatics equation (Poisson equation) in the depletion region of the pn junction. When the depletion region stores total charges of $\pm Q(V)$ for a given bias voltage V, the small-signal capacitance is given by $C = |dQ/dV|$.

(c) Consider a silicon pn junction with doping densities of $N_a = 8 \times 10^{15}/\text{cm}^3$ in the p-region and $N_d = 1 \times 10^{17}/\text{cm}^3$ in the n-region. Calculate the depletion layer depth into the p-region, depletion layer depth into the n-region, and maximum electric field strength within the depletion layer, at biases of -5 V (reverse bias), 0 V, and 0.3 V (forward bias). Plot the capacitance $C(V)$ of the pn-junction as a function of the bias voltage V, assuming that the cross-sectional area of the pn-junction is 25 μm^2. The silicon electric permittivity is $\epsilon \approx 12\epsilon_0$ where $\epsilon_0 = 8.854 \times 10^{-12}$ F/m.

(d) Consider a semiconductor pn junction in equilibrium. The conduction band electron concentration will decrease as you walk from the n-region to p-region. Therefore, one could first think of an electron diffusion current in the conduction band, which would flow from the n-region to the p-region. The magnitude of this electron diffusion current density would be expressed as

$$|J_{diff}(x)| = \left|eD_n \frac{dn(x)}{dx}\right| \tag{2}$$

where D_n is the diffusion constant of conduction band electrons, x is the spatial coordinate along the pn junction, and $n(x)$ is the conduction band electron concentration at position x. Now, the electric field in the depletion region opposes the diffusion of electrons. This may be thought of as an opposite-direction conduction band electron drift current that would cancel the electron diffusion current. The magnitude of the drift current density would be expressed as

$$|J_{drift}(x)| = |en(x)v_n(x)| = |en(x)\mu_n E(x)| \tag{3}$$

where $v_n(x)$ is the drift velocity of electrons at position x, μ_n is the mobility of electrons, and $E(x)$ is the electric field at position x. Using $|J_{diff}(x)| = |J_{drift}(x)|$ in equilibrium, derive the celebrated Einstein relation: $D_n/\mu_n = k_B T/e$.

Problem 2 (50 pt): bipolar junction transistor

Consider a p^+np bipolar junction transistor with $p+$ region called emitter, n region base, and p region collector. Draw the equilibrium energy band diagram. Also draw the band diagram when the p^+n junction is forward biased and the np junction reverse biased. Under this bias arrangement (called forward active bias), explain the transport of the charge carriers, drawing what is similar to the top figure on page 5 of Lecture #17.