Taming Electrical Solitons -
A New Direction in Picosecond Electronics

(*Invited Paper*)

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Abstract— The authors recently introduced the first electrical soliton oscillator that self-generates a periodic, stable train of electrical soliton pulses [1] - [3]. The soliton oscillator, which has been historically difficult to realize due to the soliton's instability-prone dynamics, was made possible by combining the well-known nonlinear transmission line supporting electrical soliton propagation with an amplifier that "tames" the instability-prone soliton dynamics on the nonlinear line. The ability to self-generate well-controlled solitons demonstrated in this work offers a new direction in picosecond electronics. This paper reviews this recent work and discusses future extensions.

Index Terms— Nonlinear transmission lines, electrical solitons, electrical soliton oscillators, picosecond pulse generation.

1. INTRODUCTION

The generation of electrical pulses with durations in the low picosecond range (< 10 ps) is of considerable interest in modern electronics for ultrafast time-domain metrology. This is because the short pulse duration directly translates to high temporal resolution in time-domain measurement systems. For example, the narrow electrical pulses can be used to sample, or take "snapshots" of, rapidly varying electrical signals with picosecond temporal resolution [4], [5]. For another example, the picosecond electrical pulses can be used as probe signals for high-precision time-domain reflectometry (TDR) [6].

The nonlinear transmission line (NLTL), a 1D-lattice of inductors and varactors [Fig. 1(a)], stands as one of the most powerful electronic vehicles to generate picosecond electrical pulses [5]. This is because of the NLTL's singular ability to compress an input pulse into a unique narrow nonlinear pulse known as a soliton. The active studies of the generation of narrow electrical soliton pulses on the NLTL during the past 40 years [5] have culminated in the state-of-the-art NLTL that can achieve a pulse rise time down to 480 fs [7]. In these fascinating developments, however, the NLTL has been used as a "2-port" (input + output) system, which requires an external high-frequency input to produce the narrow soliton output pulse [Fig. 1(a)].

Departing from the past 2-port NLTL work, the authors recently introduced the first "1-port" (output only) autonomous soliton oscillator that *self*-generates a periodic, stable train of electrical solitons, *not* requiring an external high-frequency input [1] - [3]. Construction of such a 1-port soliton oscillator has been historically difficult due to the oscillation instabilities caused by the solitons' nonlinear dynamics [8]. Our soliton oscillator was made possible by combining the NLTL with a unique stabilizing amplifier in a circular topology [Fig. 1(b)]. The amplifier, in addition to providing gain, "tames" the instability-prone soliton dynamics on the NLTL, allowing for stable soliton oscillation. Due to the self-selection of optimum pulse dynamics through the closed-loop operation, the 1-port system provides better pulse quality control than the 2-port NLTL. The soliton oscillator is an electrical analogue of optical soliton modelocked lasers [9]. It should be noted that our amplifier is remarkably similar in its operating properties to the amplifier developed by Cutler in 1955 for his electrical linear-pulse (*nonsoliton*) oscillator using a *linear* transmission line [10]. Due to the nonlinearity in the NLTL, however, the signal dynamics and stability issues of our soliton oscillator are fundamentally different from those of Cutler's oscillator [2].

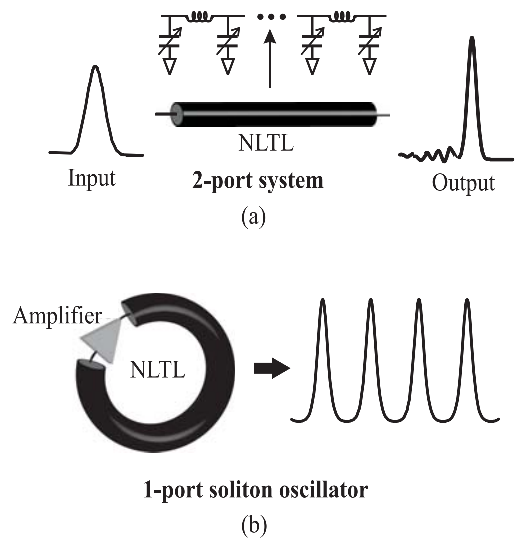

Figure 1: (a) 2-port NLTL. (b) 1-port soliton oscillator.

We have so far developed three soliton oscillator prototypes [1]-[3] with decreasing pulse width (from 43 ns to 293 ps), with the latest prototype implemented at the chip-scale. Now that the ability to self-generate well-"tamed" solitons is demonstrated, the soliton oscillator, especially its NLTL, can be quickly scaled to a smaller size and optimized to provide a much narrower pulse width. For instance, the ultrafast 2-port NLTL of [7] (480 fs pulse rise time) can be incorporated in an integrated soliton oscillator to produce a soliton width close to 1 ps. Such ultra-fast electrical soliton oscillators will offer a new platform for picosecond pulse electronics.

This paper reviews the exciting development of the electrical soliton oscillator, and discusses future extensions of the work.

2. NLTL & ELECTRICAL SOLITONS - A REVIEW -

This section briefly reviews certain unique properties of electrical solitons on the NLTL. This is to provide the necessary background to understand our soliton oscillator work [Sec. 3].

Solitons are a unique class of pulse-shaped waves that propagate in nonlinear dispersive media. They maintain spatial confinement of wave energy in a pulse shape in the course of propagation (no dispersion) and exhibit singular nonlinear dynamics [11]. Balance between nonlinearity and dispersion creates the soliton phenomena. Solitons are found in various nonlinear dispersive media throughout nature, *e.g.*, hydrodynamic solitons in shallow water and optical solitons in fiber optic cables [11].

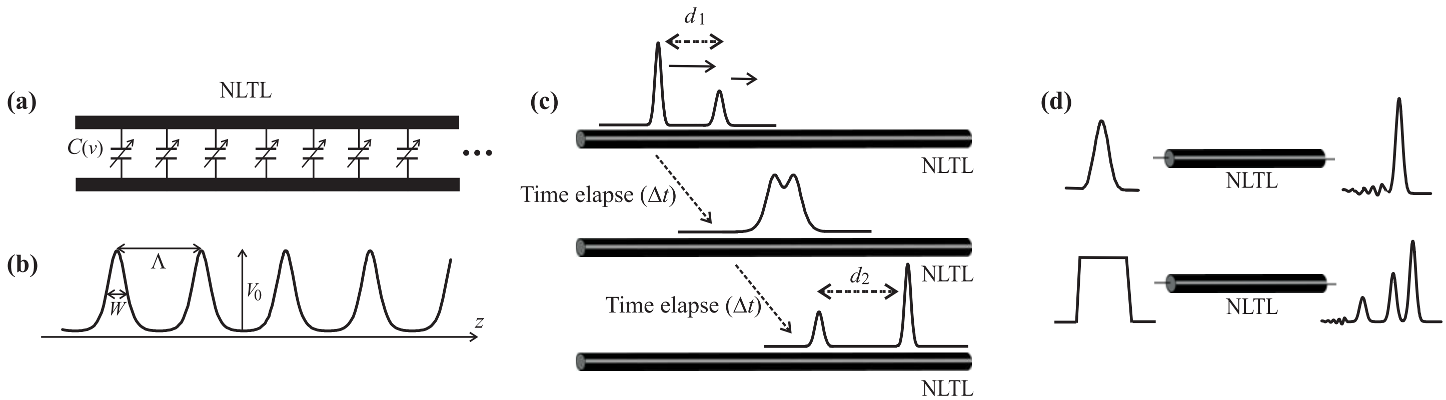

Figure 2: (a) NLTL: a linear transmission line periodically loaded with varactors. (b) A general soliton waveform. (c) Depiction of solitons' amplitude-dependent speed and nonlinear collision on an NLTL. $d_1 \neq d_2$. (d) Hypothetical transient, soliton-forming processes on the NLTL.

In the electrical domain, the nonlinear transmission line (NLTL), a 1D ladder network of inductors and varactors [Fig. 1(a)] or a linear transmission line periodically loaded with varactors [Fig. 2(a)], serves as a nonlinear-dispersive medium where electrical solitons can propagate in the form of voltage waves. In the NLTL, the nonlinearity originates from the varactors while the dispersion arises from the structural periodicity.

For certain pulse-shaped voltage waves on the NLTL, the nonlinearity balances out the dispersion, and they propagate on the NLTL maintaining their exact shape (in the absence of loss). These are electrical solitons. The general soliton propagation solution on the NLTL is a periodic train of soliton pulses (a. k. a., cnoidal wave) [Fig. 2(b)]. In the presence of loss, the solitons cannot maintain their exact shape in the course of propagation since they have to lose energy, but they still maintain spatial confinement of wave energy in a pulse shape through a unique soliton damping process [12].

In addition to their ability to maintain spatial confinement of wave energy, the electrical solitons on the NLTL possess other unique properties [11]. To begin with, a taller soliton travels faster than a shorter one on the NLTL. Due to this amplitude-dependent speed, if a taller soliton is placed behind a shorter one as shown in Fig. 2(c) top, the taller one will catch up with the shorter one and move ahead of it after a collision [Fig. 2(c)]. When two solitons collide [middle of Fig. 2(c)], they do not linearly superpose and experience a significant amplitude modulation (nonlinear collision). After the collision [bottom of Fig. 2(c)], the two solitons that have returned to their original shapes have however acquired a permanent time (phase) shift, shown by the difference in d_1 and d_2 in Fig. 2(d) (with no time shift, d_1 and d_2 would be equal since the time elapse before and after the collision is the same). The three soliton properties above, i.e., 1) *amplitude-dependent speed*, 2) *amplitude modulation during the collision*, and 3) *phase modulation after the collision*, are the key sources of soliton oscillation instabilities as will be seen [Sec. 3].

Non-soliton waves can also travel on the NLTL, but only by changing their shape to form into a soliton or solitons. A non-soliton pulse close to soliton shape will be sharpened into a soliton (Fig. 2(d), top). A non-soliton pulse that is significantly different from soliton shape will break up into multiple solitons of different amplitudes (Fig. 2(d), bottom). It is these transient soliton-forming processes that have been widely exploited in the traditional 2-port NLTL approach to generate sharp electrical pulses [5]. In our 1-port soliton oscillator design [Sec. 3], the transient process shown at the top of Fig. 2(d) is beneficial, but the process at the bottom of the figure can cause oscillation instabilities.

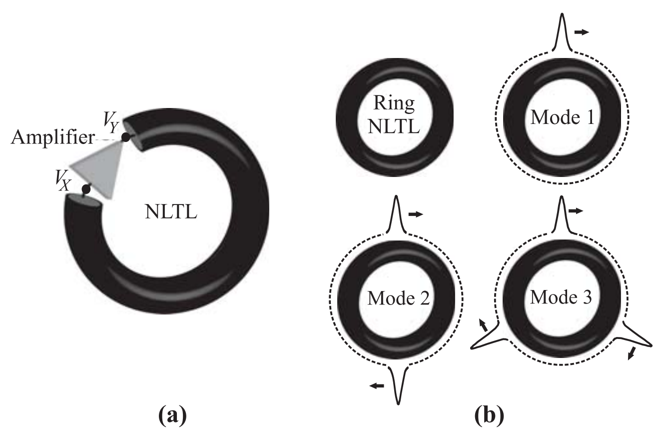

Figure 3: (a) Soliton oscillator topology. (b) Ring NLTL. Mode 1 ($l = \Lambda$), Mode 2 ($l = 2\Lambda$), Mode 3 ($l = 3\Lambda$).

3. ELECTRICAL SOLITON OSCILLATOR

3.1. Topology and Operating Principles

The starting idea to build our 1-port soliton oscillator was to combine a ring NLTL with a non-inverting amplifier inserted in the ring [Fig. 3(a)] [1], [2]. The ring NLTL supports certain soliton circulation modes determined by the periodic boundary condition, $l = n\Lambda$ ($n = 1, 2, 3, \ldots$) (l: circumference of the ring NLTL, Λ: spacing between two adjacent solitons) [Fig. 3(b)]. The amplifier provides gain to initiate an initial startup transient and to compensate loss in steady state. The ultimate goal of this topology is to self-generate and self-sustain one of the soliton circulation modes of Fig. 3(b).

The topology does indeed lead to oscillations, self-starting from noise. However, when standard amplifiers are used in the topology, the oscillations tend to be plagued with instability problems, exhibiting significant variations in pulse amplitude and repetition rate [1], [2],[8]. See Fig. 4.

The oscillation instabilities arise because the circular loop topology of Fig. 3(a) not only generates the desired soliton circulation mode, but can also excite other parasitic solitons [1], [2]. The desired and parasitic solitons continually collide while circulating in the loop due to their generally different amplitudes and resultant speed difference (due to solitons' amplitude-dependent speed, Sec. 2). It is these soliton collision events that cause the significant modulations in the pulse amplitude and repetition rate (these undesirable effects of the soliton collision were

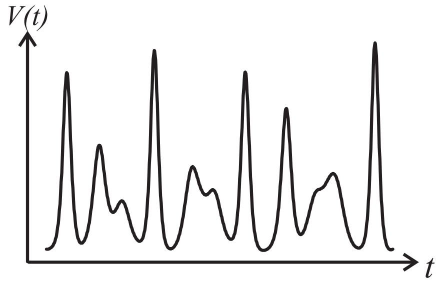

Figure 4: Unstable oscillations that can result from Fig. 3(a).

described in Sec. 2), leading to the oscillation instabilities.

The authors overcame the instability problems and obtained a stable soliton oscillator in [1] - [3] by developing a special amplifier, which not only provides gain but also incorporates three stability mechanisms to prevent the soliton collision events in steady state. The three stability mechanisms are:

Reduced signal saturation: If the amplifier saturates its output significantly in Fig. 3(a), the amplifier output will be close to a square pulse. As explained with Fig. 2(d), bottom, this square pulse will break apart into multiple solitons of differing amplitudes. These multiple solitons will circulate around the loop at different speeds (due to the amplitude-dependent speed), and be again distorted by the amplifier, creating even more solitons of different amplitudes and speeds. This process repeats itself, and the solitons continue to circulate in the loop at different speeds, continually colliding with one another, causing pulse amplitude and repetition rate variations, causing oscillation instabilities. It is therefore necessary to minimize distortion.

Perturbation rejection: In steady-state oscillation the amplifier should attenuate any small ambient perturbation (*e.g.*, noise) that could otherwise grow into parasitic solitons. Unless this is achieved, the desired soliton circulation mode and parasitic solitons will propagate at different speeds due to their generally different amplitudes, colliding and building up oscillation instabilities.

Single mode selection: The amplifier should select a single soliton circulation mode in steady-state oscillation among the many possible modes [Fig. 3(b)]. If this is not achieved, various modes with generally different amplitudes will circulate in the loop at different speeds, leading to soliton collision events and hence unstable oscillations.

In [1] - [3], we achieved these three stability mechanisms in an amplifier by incorporating an adaptive bias control in a standard saturating amplifier. Figure 5, showing the input-output transfer curve of the saturating amplifier, explains how this is achieved. The transfer curve is divided into the attenuation, gain, and saturation regions based on the curve's tangential slopes. At startup the amplifier is biased at point **A** in the gain region so that ambient noise can be amplified to initiate the oscillation startup. As the oscillation grows and forms into a pulse train, the *dc* component of the amplifier output increases. This increase in the *dc* component is used to adaptively lower the amplifier bias (dashed arrow in Fig. 5). The reduced bias corresponds to an overall gain reduction, since a portion of the pulse enters the attenuation region. The bias point continues to move down on the curve until the overall gain becomes equal to the system loss, settling at the steady-state bias **B**.

In steady state with the bias at **B**, the three stability mechanisms are simultaneously satisfied. First, the reduced bias ensures that the peak portions of the input pulses do not enter the saturation region, reducing distortion (*reduced signal saturation*). Second, with the reduced bias, the steady-state input

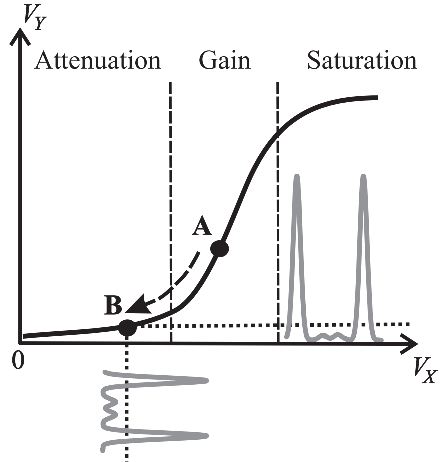

Figure 5: Transfer curve of a saturating amplifier. Startup bias **A** is in the gain region. As the *dc* component of the amplifier output increase in initial transient, the bias is adaptively lowered (dashed arrow) towards steady-state bias **B**.

soliton train is placed *across* the attenuation and gain regions, causing small perturbations around the bias to be attenuated (*perturbation rejection*). Note that perturbation rejection is accomplished while maintaining gain for the main portions of the input soliton train to compensate loss. This threshold-dependent gain-attenuation mechanism is a technique widely employed in modelocked lasers in optics, where it is known as *saturable absorption* [9], but was originally introduced in electronics domain by Cutler for his linear pulse oscillator [10]. Third, the dependence of the steady-state bias on the *dc* component of the output leads to a mode-dependant gain since each mode has a different *dc* component. This can be used to select one particular mode (*single mode selection*). See [2] for details.

3.2. Experimental Results

Three soliton oscillator prototypes have confirmed the concepts and operating principles of our soliton oscillator. The first two prototypes [1], [2] were built using discrete components (measured pulse widths: 43 ns and 827 ps) in order to explicitly examine the detailed dynamics of the soliton oscillator. The third prototype [3] was implemented on a CMOS integrated circuit (measured pulse width: 293 ps). Figure 6 shows the measured steady-state soliton oscillations from each prototype.

The most fascinating dynamics of the soliton oscillator can be observed by following the pulse around the oscillator loop in steady state. Figure 7 shows such spatial dynamics measured from our first prototype [2]. At the output of the amplifier the pulse (width: 100 ns) is not exactly a soliton and, hence, sharpens into a soliton while propagating down the NLTL. Once the soliton is formed at the eighth section (width: 43 ns), it does not further sharpen since it is now a soliton. Instead, the loss on the NLTL becomes the dominant process, and the soliton exhibits soliton damping [12] as it further travels down the NLTL, reducing its amplitude and velocity while increasing its width. At the end of the NLTL, the pulse width has increased to 110 ns. It is this clear existence of the transition point (eighth section) between two distinctively different processes, the pulse sharpening and the pulse widening, that unequivocally confirms the formation of the soliton at that transition point.

4. FUTURE EXTENSIONS

The minimum pulse width of 293 ps achieved in our latest prototype is not a record number as compared to the state-of-the-art

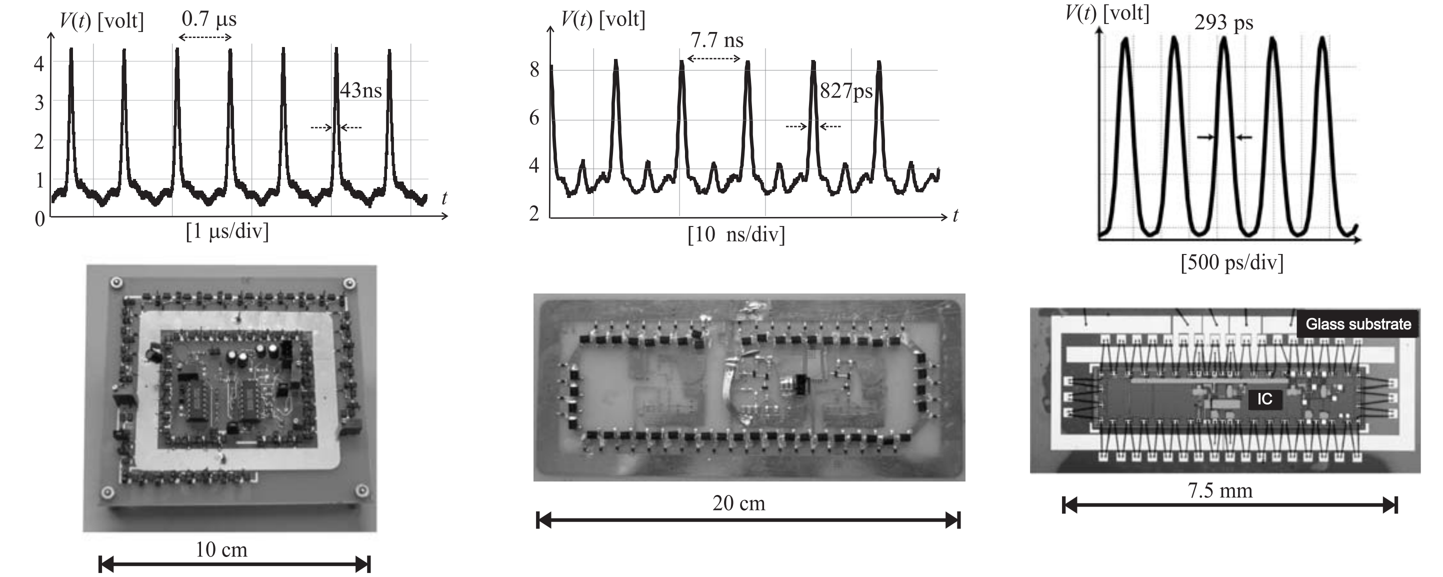

Figure 6: (Left) First soliton oscillator (pulse width: 43 ns, pulse repetition rate: 1.4 MHz). (Center) Second soliton oscillator (pulse width: 827 ps, pulse repetition rate: 130 MHz). (Right) Third, chip-scale soliton oscillator (pulse width: 293 ps, pulse repetition rate: 1.14 GHz)

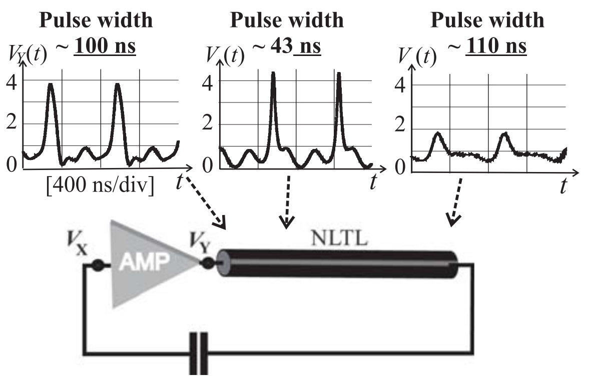

Figure 7: Measured spatial dynamics of the first soliton oscillator [2].

2-port NLTL. The value of our work so far, rather, lies in the clear demonstration of the soliton oscillator concept. Now with the soliton oscillator firmly demonstrated, it can be quickly extended to a significantly higher speed. For instance, the ultrafast 2-port GaAs NLTLs in [7] (480 fs pulse rise time) can be incorporated in our integrated soliton oscillator to substantially reduce the soliton pulse width.

Placing such an ultrafast NLTL in the electrical soliton oscillator raises an important question on the impact of the amplifier bandwidth on the minimum soliton pulse width. While the propagation of a 1-ps wide pulse on the stand-alone NLTL is feasible as demonstrated by [7], amplifiers, even in the state-of-the-art solid-state technologies, cannot provide bandwidth for such a sharp pulse. The experimental results in Fig. 7 clearly suggest, however, that the soliton compression on the NLTL may be able to overcome the bandwidth limitation of the amplifier, and hence, it may be feasible to achieve a 1-ps pulse width using the NLTL of [7] despite the relatively slower amplifier. The explicit demonstration of this interesting possibility remains an open question, and would be a natural future extension of this work.

One additional note should be made that the soliton oscillator topology of Fig. 3(a) employed in our previous works used lumped amplification. Therefore the design of a distributed soliton oscillator where the gain is provided all along the NLTL could be an important future direction to further enhance the speed. Incorporating the stability mechanisms in the distributed gain elements would be an important problem to solve.

5. ACKNOWLEDGMENT

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